Calculation of Lamb shift for states with $j = 1/2$

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The Lamb shift in the light hydrogen-like atom yields one of the most stringent test of quantum electrodynamics. It is customary to write the Lamb shift of general S-state in the form

$$\Delta E_n = \frac{n^3 \Delta E_n - \Delta E_1}{n^3} + \frac{\Delta E_1}{n^3}, \tag{1}$$

where n is the principal quantum number of the state under consideration. The first and the second terms on the right-hand side are referred to as the state-dependent and the state-independent parts, respectively. The state-dependent part of the S-states is nearly completely given by the self-energy effect in one-loop approximation which is usually expressed in the form

$$\Delta E_n = \frac{\alpha}{\pi} \frac{(Z\alpha)^4}{n^3} F(n, l_j, Z\alpha), \tag{2}$$

where l and j are the orbital and total angular quantum numbers of the state under consideration, Z is the charge of the nucleus in units of elementary charge e, $\alpha = e^2/(4\pi)$ is the fine structure constant.

It was suggested in [1,2] to calculate the dimensionless function F by means of relativistic generalization of multipole expansion (RME)

$$F = \sum_{v=1}^{\infty} F_v, \tag{3}$$

where F_v are the "relativistic multipoles" (for formulas see Eq. (73) of [2]).

Here, we report the evaluation of the state-dependent part of the S-states and $P_{1/2}$ states by means of RME. We obtain the results for $Z = 1 - 50$ and $n = 2 - 10$.

The results for low Z are the most accurate results given so far in the literature. In the case of hydrogen the uncertainty is significantly less than 1 Hz. Some of the results are presented in the following table.

Term	State	$Z = 1$	$Z = 2$	$Z = 3$	$Z = 4$
Lead	$2s - 1s$	0.229991606931	0.230390709933	0.230936267290	0.231594004269
F_3		0.000039870858	0.000154033610	0.000336061748	0.000581009986
F_4		6.241492×10^{-8}	4.634566×10^{-7}	1.469760×10^{-6}	3.298274×10^{-6}
Total		0.2300315402(3)	0.230545207(2)	0.231273799(6)	0.23217831(1)
Lead	$3s - 1s$	0.288771400893	0.289100037681	0.289541947085	0.290067435815
F_3		0.000048953133	0.000189443771	0.000413949977	0.000716666402
F_4		7.723537×10^{-8}	5.752828×10^{-7}	1.829313×10^{-6}	4.115199×10^{-6}
Total		0.2888204313(3)	0.289290057(2)	0.289957726(8)	0.29078822(2)
Lead	$4s - 1s$	0.312542251722	0.312795865525	0.313129873756	0.313519935279
F_3		0.000052358450	0.000202772029	0.000443363221	0.000768038994
F_4		8.348329×10^{-8}	6.226729×10^{-7}	1.982136×10^{-6}	4.463102×10^{-6}
Total		0.3125946937(3)	0.312999260(3)	0.313575219(8)	0.31429244(2)

Contribution of individual multipoles to the normalized difference of S-states for $n = 2 - 4$. Lead stands for first two multipoles, $F_1 + F_2$.

References

[1] J. Zamastil, V. Patkóš, Phys. Rev. A **86**, 042514 (2012).
[2] J. Zamastil, V. Patkóš, Phys. Rev. A **88**, 032501 (2013).