Description of the evolution of Rydberg systems and interaction of light with multi-level atoms using Floquet technique

D. K. Efimov\textsuperscript{1}, N. N. Bezuglov\textsuperscript{1}, and G. Juzeliunas\textsuperscript{2}

\textsuperscript{1}St. Petersburg State University, St.Petersburg, Russia
\textsuperscript{2}Institute of Theoretical Physics and Astronomy, Vilnius University, Lithuania

Presenting Author: dmitry.efimov@de29866.spb.edu

The Floquet theory treats the first order linear differential equations systems having time-dependent periodic coefficients. The Floquet’ main theorem outlines a canonical form for special fundamental solutions of a specific linear system. It is a temporal analogue of the Bloch’s theorem which describes solutions features for spatially periodic systems. The Floquet theorem indicates a canonical way to construct solutions for the evolution of a particle in time-periodic potentials induced, for instance, with monochromatic external electrical fields.

The essence of the Floquet technique is as follows. We extend the phase space for classical problems, or the Hilbert space in the quantum case by introducing the new “phase” variable $\theta$. Rewritten in the new extended spaces, the Hamiltonian equation, or Schroedinger’s equation, becomes time-independent, resulting in essential simplification of its solutions.

We consider two examples in the field of atomic physics, where the application of the technique turns out to be constructive. The first one deals with the classical treatment of a hydrogen Rydberg atom evolution under the influence of a monochromatic microwave field with the frequency $\omega$\textsuperscript{[1]}. The electron’s motion under the Floquet representation is described in $(2 \cdot 3 + 2)$-dimensional space with two additional canonical “action-angle” types variables $I, \theta$. The Floquet Hamiltonian imposes a trivial solution $\theta = \omega t$ for the angular variable. Correspondingly, there is one more non-trivial Hamiltonian equation for $I$. Such an trick allows one to provide analysis of the time-dependent problem with the use of methods which are valid only in steady-state cases.

The second example considers interaction of multilevel atoms with a set of $n$ laser fields having different frequencies. Here a standard application of the RWA is impossible, so one needs to use a smarter technique \textsuperscript{[2]}. We have constructed a new Hilbert space with extra $n$ dimensions. The modified Schroedinger’s equation in the new extended space becomes time-independent, a fact that allows one to use conventional approximations valid only for time-independent problems.

References