What is EXPERIMENTAL PHYSICS?

1. Let us start with general structure of scientific research

Full scale research process includes both fundamental and applied research activities as creative gaining of new life experience for life to satisfy definite needs of the person or society.
2. What does it mean “measurement of physical quantity”?

MEASUREMENT

General structure of measurement

**MATHEMETICAL PRESENTATION** of the results of measurements

**Numbers in Mathematics and Physics**

All numbers in Physics are approximate numbers and must be accompanied with corresponding units

Geometric presentation of numbers:

**p o i n t s** in Mathematics and **i n t e r v a l s** in Physics

**SIGNIFICANT FIGURES**

Approximate number as a result of measurement in physics must contain only significant figures. Single direct measurement made with instrument on a scale with smallest division 1 should be presented as 36 units, what means interval \((36,0 \pm 0.5)\) units. Figures after the decimal point cannot be justified.
Measurements of distance and time
Measurement of temperature

**Smallest unit** 0.1 °C

36.5 36.6 36.7

**Precision:** ± 0.05 °C

36.6 °C

(36.60 ± 0.05) °C

**Smallest unit** 0.01 °C

(35.310 ± 0.005) °C
Measurement of electric quantities
SINGLE and REPEATED measurements
\( ( \mathbf{a} , \mathbf{a}_i ) \)

INVESTIGATION OF RELATIONSHIPS
( changes and interrelations of properties – statics, kinetics, causal relationships )

Kinetics:
\[ a(t) = a_0(t_0) + \Delta a(\Delta t), \]
\[ \Delta a(\Delta t) = a(t) - a_0(t_0) = \sum_{i=1}^{n} \Delta a_i(\Delta t_i) \]
\[ a(h, g, f) \]

MATHEMATICAL PRESENTATION of the experimental data
( tables, graphs )

\begin{tabular}{c|c}
\( a \), unit & \( t \), unit \\
\hline
\( a_0 \) & \( t_0 \) \\
\( a_1 \) & \( t_1 \) \\
\( a_2 \) & \( t_2 \) \\
\( a_3 \) & \( t_3 \) \\
\( \ldots \) & \( \ldots \) \\
\( a_i \) & \( t_i \) \\
\( \ldots \) & \( \ldots \) \\
\( a_n \) & \( t_n \) \\
\end{tabular}

MATHEMATICAL MODELLING of observed relationships

Mathematical Physics

Fund.research of physical phenomena

Applied research of physical phenomena

Experimental Physics
3. Precision (accuracy, uncertainty) of measurement result –

truth and real values of measured properties/quantities,

systematic, random and total error of measured quantities

Error – characteristic of measured quantity precision

SYSTEMATIC ERRORS

A reading consistently shifted in one direction is called a systematic error. Examples include: a zero error on any scale, a calibration error, a background count in a radioactivity experiment, an end correction in a resonance tube, a stray magnetic field, backlash on screw threads. Some systematic errors can be corrected either by adjustment of the instrument or by noting the error and correcting all readings appropriately. Systematic errors are the more serious form of error since they cannot be reduced by taking repeated readings or by any other form of averaging.

Treatment of systematic errors

Fig. a shows a spread of readings caused by random errors; these are approximately centred about the true value. If a systematic error is also present these readings will be shifted so they are no longer centred about the true value but about some other value (Fig. b). Under these experimental conditions, no matter how many readings are taken, the final result will not approach the true value.

![Random errors](image1)

Random errors distributed about the true value.

![Systematic error](image2)

Random errors superimposed on a systematic error.
Treatment of random errors

When certain quantities are measured, the measured values are known only to within the limits of experimental uncertainty. The value of the uncertainty can depend on various factors, such as the quality of the apparatus, the skill of the experiment, and the number of measurements performed.

In general, experimental uncertainty consist on two terms: uncertainty of the apparatus $\Theta$, and uncertainty $\varepsilon$ caused by random variance of measured data from the real value of quantity. First parameter can be found in the apparatus user's manual or assumed to be smallest unit on the apparatus scale. The second can be estimated by analyzing of statistical properties of measured data set. Under normal conditions the probability of occurrences of value $\varepsilon$ corresponds to Gaussian distribution.

![Gaussian distribution](image)

There $a$ is real value of quantity and $\sigma$ - dispersion that measures how widely measured values are dispersed from the real value of quantity. Two basic statistical parameters of data set $x_i$ are average value $\bar{x}$ and mean quadratic deviation $S$, that are defined using following mathematical expressions

$$\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i, \quad S = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n-1}} = \sigma$$

As the number of performed measurements increases ($n \to \infty$), average value $\bar{x}$ tends to the real value of quantity $\bar{x} \to a$, and mean quadratic deviation $S$ tends to the dispersion $S^2 \to \sigma^2$. The estimation of the limits of random perturbation caused uncertainty usually have been based on the property that the interval $[\bar{x} - \varepsilon, \bar{x} + \varepsilon]$ must contains 95% of all measurement results. Theoretical calculations indicated that for large number of experiments $\varepsilon = 2.78S(\bar{x})$. There $S(\bar{x})$ is statistical quantity called mean quadratic deviation of mean arithmetic value, defined by expression

$$S(\bar{x}) = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n(n-1)}}$$

The total experimental uncertainty $\Delta x$ in direct measurement is represented by expression

$$\Delta x = \sqrt{\Theta^2 + \varepsilon^2}$$
Generally the contribution of an error can be neglected if the error is less than about 1/10 of the dominant or total error. If systematic error is approximately the same value as random error, **systematic error 0 and maximum random error ε** must be presented.

Maximum random error ε can be calculated using Student’s coefficient $t_{n, \gamma}$, where $n$ is number of repeated measurements and $\gamma = 0.95$

$$\varepsilon = t_{0.95, n} \times \sqrt{\left[ \sum (d_a - d_i)^2 \right] / \left[ n \times (n-1) \right]}$$

<table>
<thead>
<tr>
<th>n</th>
<th>$t_{0.95, n}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>4.303</td>
</tr>
<tr>
<td>4</td>
<td>3.182</td>
</tr>
<tr>
<td>5</td>
<td>2.776</td>
</tr>
<tr>
<td>6</td>
<td>2.571</td>
</tr>
<tr>
<td>7</td>
<td>2.447</td>
</tr>
<tr>
<td>8</td>
<td>2.365</td>
</tr>
<tr>
<td>9</td>
<td>2.306</td>
</tr>
</tbody>
</table>

**Total error of the result of measurement means account of both - systematic and random errors**

Direct measurement of property $a$

<table>
<thead>
<tr>
<th>Basic types of errors</th>
<th>Absolute error</th>
<th>Relative error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Systematic error</td>
<td>$\Delta a_s$, unit</td>
<td>$\delta a_s = (\Delta a_s/a) \times 100%$</td>
</tr>
<tr>
<td>Random error</td>
<td>$\Delta a_r$, unit</td>
<td>$\delta a_r = (\Delta a_r/a) \times 100%$</td>
</tr>
<tr>
<td>TOTAL error</td>
<td>$\Delta a = \sqrt{(\Delta a_s)^2 + (\Delta a_r)^2}$, unit</td>
<td>$\delta a = (\Delta a/a) \times 100%$</td>
</tr>
</tbody>
</table>

Result of the measurement of definite physical quantity - property $a$ is presented as approximate number: $a$ (units) alone or with its total absolute error $(a +/- \Delta a)$ units and its relative (fractional) error $\delta a = (\Delta a/a) \times 100\%$

If there is approximate number $a$ (unit) alone $[a = 253$ units$]$, it contains only significant figures what depend on total precision (total error) of definite measurement. If total error also is presented, total error contains not more than two significant numbers $[a = (253, 12 +/- 0.25 )$ units, where 0.25 is calculated total absolute error of calculated quantity 253,12$].

**DIRECT and INDIRECT measurements**

<table>
<thead>
<tr>
<th>Measurements</th>
<th>DIRECT</th>
<th>INDIRECT</th>
</tr>
</thead>
<tbody>
<tr>
<td>SINGLE</td>
<td>$a$</td>
<td>$a (b, c, d ....)$</td>
</tr>
<tr>
<td>REPEATED</td>
<td>$a_i (t_i) = \text{const}$</td>
<td>$a (b, c, d ....) = \text{const}$</td>
</tr>
<tr>
<td></td>
<td>$a_i (t_i) \neq \text{const}$</td>
<td>$a_i (t_i) = a_i (b, c_i (t_i)) \neq \text{const}$</td>
</tr>
</tbody>
</table>

Precision of indirect measurement depends on corresponding precision of direct measurements - every direct measurement comes with its total error what is producing corresponding uncertainty of indirect measurement. Determination of its total precision can be quite difficult task and special methods are used. The simplest method is corresponding adding of total relative errors of particular direct measurements.
Determination of density
[ case study of solid state material what is formed as cylindrical body ]
\( \rho \ (g/cm^3) = ? \)

<table>
<thead>
<tr>
<th>Measurement procedure consists:</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) of <em>direct measurement</em> of diameter ( d ) (cm), height ( h ) (cm) and mass ( M ) (g) of given body what include also determination of the precision of measured quantities;</td>
</tr>
<tr>
<td>2) following final <em>indirect measurement</em> - calculation of density ( \rho \ (g/cm^3) ) what again includes determination of the precision of provided indirect measurement.</td>
</tr>
</tbody>
</table>

1. **Direct measurement** of diameter \( d \) (cm) and determination of the total error

<table>
<thead>
<tr>
<th>No of measurement</th>
<th>Measured values of diameter ( d_i ) (cm)</th>
<th>( d_{av} - d_i )</th>
<th>( (d_{av} - d_i)^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (first)</td>
<td>( d_1 = )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 (second)</td>
<td>( d_2 = )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>( d_3 = )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>( d_5 = )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>.... ( etc.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totally: ( n ) measurements</td>
<td>Calculated average value ( d_{av} = ... )</td>
<td>( \Sigma (d_{av} - d_i)^2 = )</td>
<td>( = ........... )</td>
</tr>
</tbody>
</table>

**Random absolute error**

\[
\Delta d_r = t_{0.95, n}* \sqrt{\left[ \Sigma (d_{av} - d_i)^2 \right] / [n * (n-1)]} = ........... \ (cm)
\]

\( n \) - total number of measurements; \( \gamma = 0.95 \) \( t_{\gamma, n} \) - Student’s coefficient

**Systematic absolute error** \( \Delta d_s = ........... \ (cm) \)

**Total absolute error** \( \Delta d = \sqrt{\Delta d_s^2 + \Delta d_r^2} = ........... \ (cm) \)

Approximate number containing two significant figures!

**Total relative error** \( \delta d = (\Delta d / d_{av}) \times 100\% = ........ \ % \)

**Result**: \( (d_{av} \ +/- \Delta d) \ cm , \delta d \ % \)

..................................................
2. **Direct measurement** of **height** $h$ (cm) and determination of the total error

<table>
<thead>
<tr>
<th>No of measurement</th>
<th>Measured values of height $h_i$ (cm)</th>
<th>$h_{av} - h_i$</th>
<th>$(h_{av} - h_i)^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$h_1$ =</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>$h_3$ =</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>$h_5$ =</td>
<td></td>
<td></td>
</tr>
<tr>
<td>…. (etc.)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Totally: $n$ measurements</td>
<td>Calculated average value $h_{av}$ =</td>
<td>$\sum (h_{av} - h_i)^2$ =</td>
<td></td>
</tr>
</tbody>
</table>

**Random absolute error**

$$\Delta h_r = t_{0.95} \times \frac{\sqrt{\sum (h_{av} - h_i)^2}}{n \times (n-1)} = \text{…… (cm)}$$

**Systematic absolute error** $\Delta h_s = \text{……… (cm)}$

**Total absolute error**

$$\Delta h = \sqrt{\Delta h_s^2 + \Delta h_r^2} = \text{…… (cm)}$$

**Total relative error**

$$\delta h = \left( \frac{\Delta h}{h_{av}} \right) \times 100\% = \text{…… (decimal fraction)}$$

**Result:** $(h_{av} \pm/\Delta h)$ cm , $\delta h = \text{….}$ %

3. **Direct measurement** of **mass** $M$ (g) and determination of the total error

Using digital mass meter we got $M = \text{…… (g)}$

Systematic absolute error of this measurement is characterized by a half of instrument’s scale smallest division: $\Delta M_s = \text{…… (g)}$

There was no random error effect observed when repeating mass measurements – so total error of direct measurement contains only systematic error $\Delta M = \Delta M_s$

**Result:** $(M \pm/\Delta M)$ g , $\delta M = (\Delta M/M) \times 100\%$

**Final indirect measurement - calculation of density** $\rho$ (g/cm$^3$)

and determination of the precision of corresponding indirect measurement.

$$\rho = \frac{4 \times M}{\pi \times d^2 \times h} = \text{………… (g/cm}^3\text{)}$$

**Total relative error**

$$\delta \rho (\%) = \delta M (\%) + 2 \times \delta d (\%) + \delta h (\%) = \text{…… (\%)}$$

$$\delta \rho = \text{……… (decimal fraction)}$$

**Total absolute error**

$$\Delta \rho = \rho \times \delta \rho = \text{…… g/cm}^3\text{)}$$

**Final Result:** $(\rho \pm/\Delta \rho)$ g/cm$^3$ , $\delta \rho (\%)$

..........................................................
4. Graphical investigation of relationships

One of the most important reasons for experimental work is to investigate the relationships between two or more physical quantities. Taking a single set of readings of a physical quantity often gives very little information on the form of the relationship between the readings and the limitations of their validity. A single set of readings could
be used to calculate the physical constant involved; for example, measuring the potential difference $V$ across and the current $I$ through a device will give a value of the resistance $R$ of the device for those particular values of $V$ and $I$, but it would not tell you anything about the device or whether $R$ depends on $I$. A single pair of values of $V$ and $I$ (marked as +) could be interpreted in many ways (Fig. 3.1).

Clearly several sets of readings of $V$ and $I$ would enable a distinction between Figs. 3.1a, b and c to be made. (A graph is easier to interpret than a table of values of $V$ and $I$.) There is another important reason for using several sets of readings of $V$ and $I$, namely, experimental error. There are random errors which relate to the precision with which an instrument can be read, concerning the inability of the experimenter to record an exact value. Repeated measurements of the same quantity will give different values. This type of error can be reduced by taking many readings and averaging them; but a graph can also be used to achieve the same end (Fig. 3.2). There will always be a deviation of experimental points from the straight line. The straight line should pass symmetrically between the points. The slope or gradient of the line $\Delta V/\Delta I = AC/BC$ gives an average value of $R$. 

![Diagram](image-url)

**Averaging random errors graphically**
Plotting a graph

a) Always plot a graph as the experiment proceeds. This will help to detect errors and deviations.

b) Decide on the range of values of each variable which will occur in the experiment. This may sometimes involve doing a quick preliminary experiment to find maximum and minimum values.

c) If an intercept (point where the graph cuts an axis) is not required, the origin need not be included. This is often useful if the values of one quantity cover only a small range.

d) It is usually a good thing to spread the points to fill the graph. For example, in Fig. a it would be difficult to decide whether $R$ were linearly related to $T$ but it should be evident from Fig. b.

![Graphs showing proportional and linear relationships](image)

Proportional / linear relationships

A proportional relationship takes the form $y = kx$. If we plot $y$ against $x$ the graph will be a straight line through the origin.

$$\text{slope} = k = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{(y_2 - y_1)}{(x_2 - x_1)}$$
**Elastic deformation of metallic string**

[case study of elastic deformation (stretching, compression) of solid bodies - determination of Young modulus]

\[ Y \, (N/m^2) = ? \]

The absolute increase of string’s length \( \Delta L \) when stretching force \( F \) is applied depends on the amount of force applied to stretch a rod \( F \), cross-sectional area of the rod \( A \) and the nature of rod’s material.

\[ \Delta L = \frac{L_o}{Y A} \]

**Mathematical model of the elastic behaviour - deformation** for a given string can be expressed by the linear relation, if the amount of stretching is small compared to the original length of the string:

\[ \Delta L (m) = k \times F \]

where \( k = \frac{L_o}{(Y \times A)} \) means the slope of function’s graph and \( Y \) is a constant, called **Young modulus**, what value depends on the nature of the string’s material.

\[ \Delta L = \frac{L_o}{(k \times A)} \]

\[ Y = \frac{L_o}{(k \times A)} \]
Experimental Physics

Measurement of bodies' physical properties - determination of corresponding physical quantities

Deformation of solid bodies
Study of elastic deformation by streching metallic string, determination of Young modulus

$L_0 = \ldots \text{(m)}$
Initial length of given string

Mathematical model of the elastic deformation of a string corresponds to observed fact that change of string's length is directly proportional to streching force:

$$\Delta L(F) = k F = \left[ L_0 / (Y \times A) \right] F$$

It's called Hooke law, where $A$ is cross-sectional area of string and $Y$ is a constant physical quantity as characteristic of string's material - Young modulus.

$\Delta L(F)$

$F$

$Y = L_0 \times F / (A \times \Delta L)$

Change of length
$\Delta L(F) = \ldots \text{(m)}$

Streching force $F$ (N)
Direct measurements of metallic string

\[ L_0 = \ldots \text{ (mm)} = \ldots \times 10^{-3} \text{ m} \]
\[ D = \ldots \text{ (mm)} = \ldots \times 10^{-3} \text{ m} \]

**Experiment: stretching of metallic string**

\[ A = \pi D^2 / 4 = \]
\[ = 0.785 D^2 = \]
\[ = \quad = \quad (\text{m}^2) \]
\[ F (\text{N}) = F_{\text{gravitational}} = \]
\[ = M (\text{kg}) \cdot 10 \text{ (m/s}^2) \]

<table>
<thead>
<tr>
<th>M (kg)</th>
<th>( \Delta L ) (m)</th>
<th>F (N)</th>
<th>( \Delta L ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_0 ) = 0</td>
<td>( \Delta L_0 = 0 )</td>
<td>( F_0 = 0 )</td>
<td>( \Delta L_{18} = \ldots \times 10^{-3} )</td>
</tr>
<tr>
<td>( M_1 = 0.5 )</td>
<td>( \Delta L_1 = \ldots \times 10^{-3} )</td>
<td>( F_1 = 5 )</td>
<td></td>
</tr>
<tr>
<td>1.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.5</td>
<td>( \Delta L_9 = \ldots \times 10^{-3} )</td>
<td>( \Delta L_9 = \ldots \times 10^{-3} )</td>
<td></td>
</tr>
</tbody>
</table>

Determination of Young’s modulus using data from graph \( \Delta L(F) = k \times F \)

\[ k = \Delta L (\text{m}) / F (\text{N}) = \ldots \ldots \ldots \]

\[ Y = L_0 / (k \times A) = \ldots \ldots \ldots \ldots = \]
\[ = \ldots \ldots \ldots \text{ (N/m}^2 \]

Random experimental quantities of linear function’s slope are averaged graphically

Teflon - \( 3.7 \times 10^8 \) N/m\(^2\)
Nylon - \( 3.7 \times 10^9 \) N/m\(^2\)
Aluminium - \( 0.69 \times 10^{11} \) N/m\(^2\)
Copper - \( 1.1 \times 10^{11} \) N/m\(^2\)
Bone (stretching) - \( 1.6 \times 10^{10} \) N/m\(^2\)
Bone (compression) - \( 0.94 \times 10^{10} \) N/m\(^2\)
\[ \Delta L = 0.41 \, mm \]
\[ F = 20 \, N \]
Non-linear relationships

Fig. a probably represents a linear relationship – the points are spaced either side of the line drawn. In Fig. b this is not so – remember the main reason for the spread of points is random errors. Fig. b shows systematic deviations from the straight line drawn and a curve is probably a more accurate way of relating $R$ and $\theta$ for the device.

The purpose of this section is to look at particular relationships and decide what form the final graph should take if it is to be a straight line. This will enable us to see whether the relationship is valid for our experiment and, more importantly, find the constants of the equation from the slope and the intercept.
Many experimental relationships are not linear, that is a graph of \( y \) against \( x \) will be a curve.

Clearly the exact relationship could be worked out from the experimental curve, but it is difficult to distinguish between different types of curve over a small range of values and it is not easy to determine the constants from a curve. When plotting a graph during an experiment always plot the variables as measured and then decide whether the graph should be a curve or a straight line.

**Exponential relationships**

These have the form \( y = a \exp(kx) \) or \( y = ae^{kx} \).

Usually in physics this type of relationship is met in decay, when \( k \) is negative.

![Exponential decay and growth graphs](image)

To produce a linear relationship for plotting we have to take logarithms to base \( e \), (see footnote on page 23) which gives:

\[
\log e y = \log e a + kx
\]

or:

\[
\ln y = \ln a + kx
\]

Comparison with \( y = mx + c \) shows that a graph of \( \log e y \) against \( x \) should give a straight line of slope \( k \) and with intercept on the \( (\log e y)\)-axis of \( \log e a \).

**Obtaining a linear relationship for \( y = a \exp(kx) \)**

![Diagram showing method of obtaining linear relationship](image)
Exponential relationships

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To produce a linear relationship for plotting we have to take logarithms to base \( e \), (see footnote on page 23) which gives:

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Comparison with \( y = mx + c \) shows that a graph of \( \log_e y \) against \( x \) should give a straight line of slope \( k \) and with intercept on the \((\log_e y)\)-axis of \( \log_e a \).

Obtaining a linear relationship for \( y = a \exp(kx) \)

An example of this kind of relationship is the variation of the height of water flowing down a burette with time: \( h = h_0 \exp(-\lambda t) \). This can be written as:

\[
\log_e h = -\lambda t + \log_e h_0
\]

hence the slope of the graph is \(-\lambda\):

\[
-\lambda = \frac{(\log_e h_2 - \log_e h_1)}{(t_2 - t_1)}
\]

The variation in the height \( h \) of a liquid column with time \( t \)
The time taken for the water to reach half of its original height is:

\[ t_{\frac{1}{2}} = \frac{\log_e 2}{\lambda} = \frac{0.693}{\lambda} \]

Another example is the variation of DC electric current and voltage when charging/discharging capacitor.

**Direct (DC) electrical current** \( I(t) \)
**when charging/discharging capacitor**

\[
V_{R} = I \times R
\]

\[
V_C(t) = \frac{q(t)}{C}
\]

\[
I_{ch}(t) = \frac{V}{R} \exp\left(-\frac{t}{RC}\right)
\]

\[
I_{disch}(t) = -\frac{V}{R} \exp\left(-\frac{t}{RC}\right)
\]

If \( R=50k\Omega, C=100\mu F \), then \( \tau = 500s \approx 8\text{min} \)

If \( V=5V \), then \( I_{max} = 100 \mu A \)

\[
q(t) = C \times V_C(t)
\]

This is mathematical model of directly invisible physical – electric phenomena. We can directly observe only formal characteristics of the flow of electric charge: electric current and corresponding voltages.
5. Organization and realization of scientific experimental research

Introduction (what and why we are going to investigate?)

Theoretical background of research

Practical activities (organization and providing of measurements, data processing – getting final results)

Discussion of results

Conclusions (do we have reached the goal of provided research?)

Report / written and/or oral presentation of results